
## Generic Legacy Signature w/ EC keys

- private key $x$ → public key $X \triangleq xG$
- transaction **tx** to be signed
- private key $x$ + transaction **tx** → Signing Algorithm → **Signature**

Verifying Algorithm $f$(X, tx, Signature) → success ? failure

## Non-interactive (Fiat-Shamir) Schnorr

- $x$ → $X \triangleq xG$
- transaction **tx** to be signed
- $\alpha$ → $c \triangleq H(tx, \alpha G)$

*random and unique for each signature, otherwise privkey could be leaked from response r:*

*known: $x = (\alpha - r)/c$*

*reused: $x = (r_1 - r_2)/(c_2 - c_1)$*

*called **challenge** because it's known to the signer only after choice of $\alpha$ (being the output of a one-way hash involving $\alpha$), as in interactive Schnorr proof where it's provided by the verifier only after knowing $\alpha G$ (if not, in that case the signer could lie about knowledge of $x$ opportunistically choosing $\alpha$ and r)*

$r \triangleq \alpha - cx$

*called **response** because it's the signer's "answer" to previous challenge c*

Signature

$$f(X, tx, c, r)\ \left\{\ H(tx, \underbrace{rG + cX}_{=\alpha G \text{ if signature is ok}}) \overset{?}{=} c\right.$$

## Multi keys (& bases) n.i. Schnorr (i=1,…,n)

- $x_i$ → $X_i \triangleq x_i G_i$
- transaction **tx** to be signed
- $\alpha_i$ → $c \triangleq H(tx, \alpha_1 G_1, \dots, \alpha_n G_n)$

*please note that while all other elements are indexed by means of i, **challenge** is unique per multi-signature (a property someway preserved in Rings' structures)*

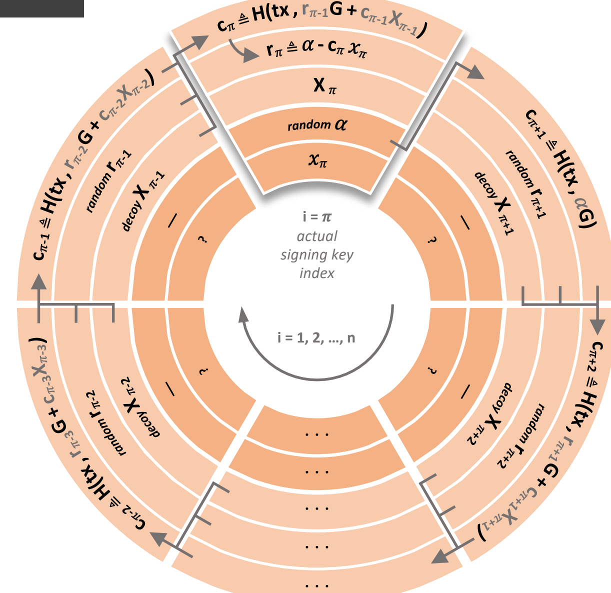
$r_i \triangleq \alpha_i - cx_i$

Signature

$$f(X_i, tx, c, r_i)\ \left\{\ H(tx, r_1G_1 + cX_1, \dots, r_nG_n + cX_n) \overset{?}{=} c\right. \quad \text{(it commits to n signatures at the same time)}$$

Rings "magic" is about finding flavours of previous schemas with decoys, while still retaining just only one ACTUAL signer (from a technical point of view: needing many $X_i$ in verifying algo but single $x$ in signing algo); and all without coordination between involved keys owners

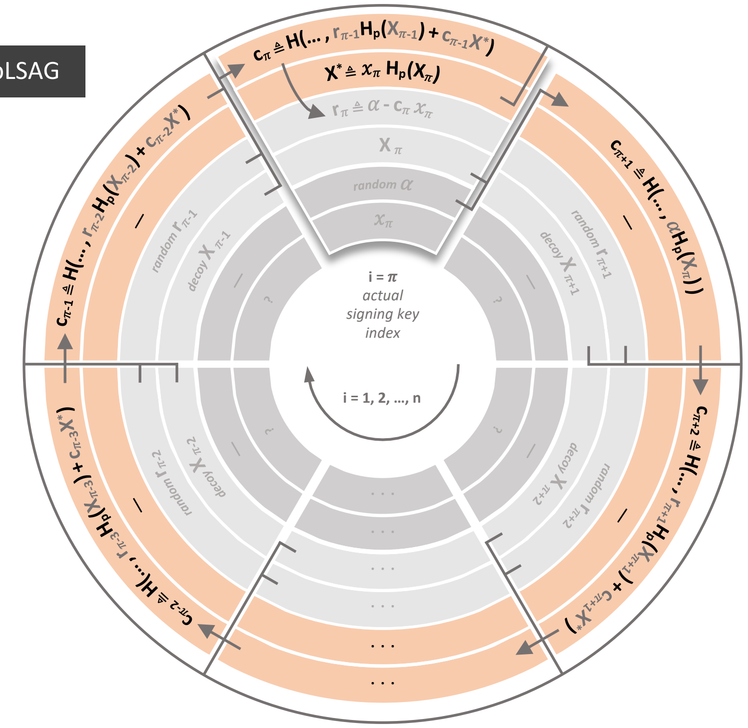
## SAG




$$c_2 = H(tx, r_1G + c_1X_1)\quad c_3 = H(tx, r_2G + c_2X_2)\quad \dots\quad c_n = H(tx, r_{n-1}G + c_{n-1}X_{n-1})\quad H(tx, r_nG + c_nX_n) \overset{?}{=} c_1$$

$$f(X_i, tx, c_1, r_i)$$

## bLSAG



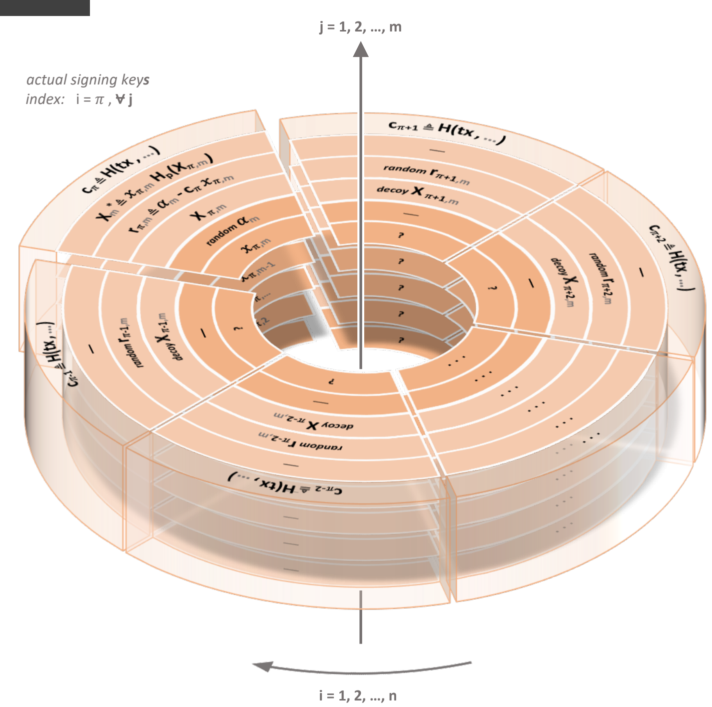

$$c_{\pi+1} \triangleq H(tx, \alpha G, \alpha H_p(X_\pi)) \qquad c_i \triangleq H(tx, r_{i-1}G + c_{i-1}X_{i-1}, r_{i-1}H_p(X_{i-1}) + c_{i-1}X^*)$$

$f(X_i, tx, c_1, r_i, X^*)$:

- $X^*$ never seen on-chain before ?
- $l X^* \overset{?}{=} 0$
- $H(tx, r_nG + c_n(tx, c_1, r_{i \neq n}, X_{i \neq n}) X_n, r_nH_p(X_n) + c_n(tx, c_1, r_{i \neq n}, X_{i \neq n}) X^*) \overset{?}{=} c_1$

  ($c_n = c_n(c_{n-1}(\dots(c_2(c_1))))$ *as previously seen in SAG*)

## MLSAG




$$c_{\pi+1} \triangleq H(tx,\ \alpha_m G,\ \alpha_m H_p(X_{\pi,m}),\ \dots,\ \dots,\ \alpha_1 G,\ \alpha_1 H_p(X_{\pi,1}))$$

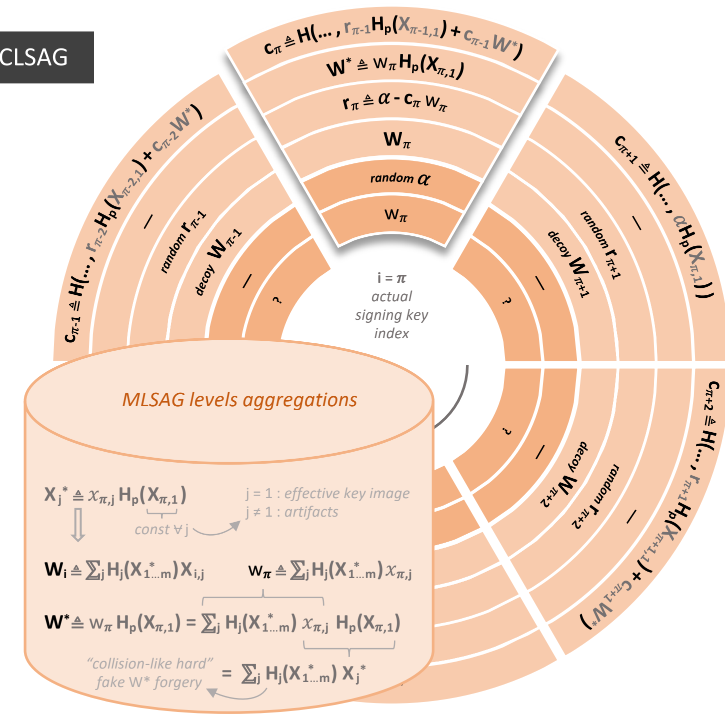
$$c_i \triangleq H(tx,\ r_{i-1,m}G + c_{i-1}X_{i-1,m},\ r_{i-1,m}H_p(X_{i-1,m}) + c_{i-1}X_m^*,\ \dots,\ \dots,\ r_{i-1,1}G + c_{i-1}X_{i-1,1},\ r_{i-1,1}H_p(X_{i-1,1}) + c_{i-1}X_1^*)$$

$f(X_{i,j}, tx, c_1, r_{i,j}, X_j^*)$:

- $X_j^*$ never seen on-chain before ?
- $l X_j^* \overset{?}{=} 0$
- $H(tx, r_{n,1}G + c_nX_{n,1}, r_{n,1}H_p(X_{n,1}) + c_nX_1^*, \dots, r_{n,m}G + c_nX_{n,m}, r_{n,m}H_p(X_{n,m}) + c_nX_m^*) \overset{?}{=} c_1$

  ($c_n = c_n(c_{n-1}(\dots(c_2(c_1))))$ *as previously seen in SAG and bLSAG*)

## CLSAG




$$c_{\pi+1} \triangleq H(tx, \alpha G, \alpha H_p(X_{\pi,1})) \qquad c_i \triangleq H(tx, r_{i-1}G + c_{i-1}W_{i-1}, r_{i-1}H_p(X_{i-1,1}) + c_{i-1}W^*)$$

$f(X_{i,j}, tx, c_1, r_i, X_j^*)$:

- $X_1^*$ never seen on-chain before ?
- $l X_1^* \overset{?}{=} 0$
- $H(tx, r_nG + c_nW_n(X_j^*, X_{n,j}), r_nH_p(X_{n,1}) + c_nW^*(X_j^*)) \overset{?}{=} c_1$

  (*again:* $c_n = c_n(c_{n-1}(\dots(c_2(c_1))))$)

# Rings unleashed notes

## SAG (Spontaneous Anonymous Group)

- the index value of actual signer ($\pi$) is random, otherwise $X_\pi$ could be deduced from the order of parameters provided in signature;
- the challenges $c_i$ are built from previous slice elements, with dependencies depicted by the arrows;
- final $r_\pi$ definition guarantees the dependencies applying to all other $c_i$ still apply to $c_{\pi+1}$ as well (even if originally calculated from $\alpha$), so challenges form a closed chain, a ring: that's why it's enough to provide $c_1$ in signature (it's the "someway preserved" single-challenge-per-multi-signature property)

## bLSAG (Back's Linkable SAG)

- bLSAG is a SAG extended with a key image $X^*$ (to prevent double spending while still mantaining anonymity, introducing linkability of signatures) and modified challenges $c_i$ to commit to that key image as well;
- $H_p(X_\pi)$ is a carefully chosen function returning a random point in EC basepoint-subgroup of prime-order $l$, acting as generator point for key image $X^* \triangleq x_\pi H_p(X_\pi)$

**some BAD key image generators**

**$H_p(X_\pi) \triangleq n(X_\pi)\,G$**
$\Longrightarrow X^* \triangleq x_\pi n(X_\pi) G = n(X_\pi) x_\pi G = n(X_\pi) X_\pi$
so actual signer could be found by tries

**$H_p(X_\pi) \triangleq G_2$**
$\Longrightarrow X_1^* \triangleq x_{\pi,1} G_2 \qquad X_2^* \triangleq x_{\pi,2} G_2$
$\Longrightarrow X_1^* - X_2^* = (x_{\pi,1} - x_{\pi,2})\, G_2$
but a previous payer to both $X_{\pi,1}$ and $X_{\pi,2}$ can calculate the value between brackets (thanks to Diffie-Hellman-like exchange at the base of Stealth Addresses), so owns heuristics to pair future $X_{\pi,1}$ and $X_{\pi,2}$ usages

**$H_p(X_\pi) \triangleq X_\pi \triangleq x_\pi G$**
$\Longrightarrow X_1^* - X_2^* = (x_{\pi,1}^2 - x_{\pi,2}^2)\, G$
like in previous case, just a bit more algebra and need to use G to get rid of remaining private spending key in favour of public one

- $l X^* = 0$ check in verifying algorithm is needed to avoid double spending due to key image "malleability". In challenges we have:

  $$c_i = H(\dots c_{i-1} X^*)$$

  however $X^*$ could be substituted by a fake $X^* + P_h$ -where $P_h$ is a point in EC subgroup of order h, the cofactor- if the attacker found (by tries) all $c_i$ multiples of h; in that case:

  $$c_i(X^* + P_h) = c_iX^* + c_iP_h = c_iX^*$$

  because any point multiplied by its subgroup order gives zero. Luckily $l(X^* + P_h) \neq 0$ because, being prime, $l$ cannot be a multiple of h

## MLSAG (Multilayer Linkable SAG)

- MLSAG is a stack of many bLSAG, with per-slice challenges $c_i$ (so one single challenge for each 3D slice, commiting to all layers);
- even if it doesn't appear to be a schema requirement, in Monero the index value of actual signer ($\pi$) is intended to be random but shared among all layers, offering inter-levels clustering opportunity to an attacker making an educated guess about actual keys: that's why multi-input transactions (where maximum savings could be attained) have preferred to avoid the use of just one single MLSAG

## CLSAG (Concise Linkable SAG)

- the schema currently used by Monero, it's a bLSAG for "pseudo keys" $w_\pi$ and $W_i$ obtained aggregating keys on MLSAG different levels; it provides back-compatible linkability (meaning usual key image generation) only for $X_{\pi,1}$;
- $W^*$ doesn't really prevent double spending by itself but it's built from effective $X_1^*$ and $X_{i\neq1}^*$ artifacts (that's why they are the ones actually used in verifying algorithm)

## Credits

This cheatsheet is deeply inspired by Zero to Monero: 2nd Edition (especially chapters 2 and 3 and mentioned sources): the notation is only slightly different and with "minor" omissions to focus on gradual presentation of Rings' core properties (e.g., no key prefixing or domain separation for hashes)